

## Quest for Mathematics I (E2): Exercise sheet 5

1. Find the antiderivatives of the following functions:

(a)  $\frac{x^3}{(1+x^2)^3}$ ;

(b)  $|x|$ ;

(c)  $\frac{e^{2x}}{(e^x+1)^2}$ ;

(d)  $x \sin x$ .

2. (a) Suppose an arch-shaped door is described by the area enclosed by a parabola  $y = A - Bx^2$  and the  $x$ -axis (where  $A, B > 0$ ). If the length of the base of the door is  $b$ , and the height is  $h$ , compute the area of the door. See Fig. (i).

(b) By integration, find the area enclosed by the circles described by the equations  $x^2 + y^2 = 1$  and  $x^2 + (y - 1)^2 = 1$ . See Fig. (ii). *Hint: You might want to use the substitution  $x = \sin u$  in the computation.*

(c) By integration, find the area enclosed by the curves  $y = 2/x$ ,  $y = x^2$  and the lines  $x = 1$ ,  $x = 2$ , as shown by the shaded area on Fig. (iii).

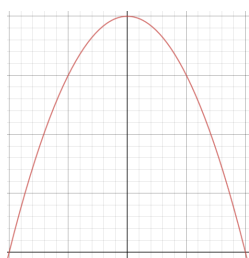


Fig. (i)

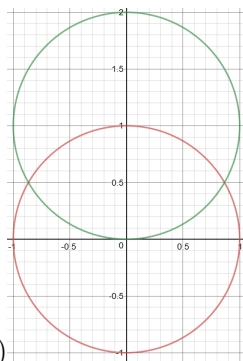


Fig. (ii)

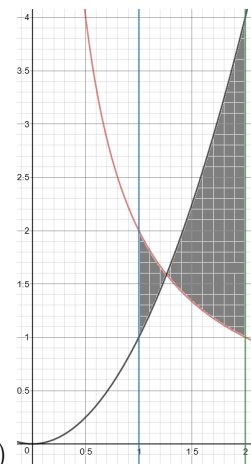


Fig. (iii)

3. Suppose a speeding car is travelling along a straight road at  $40\text{ms}^{-1}$ . As it passes, an initially stationary police car starts to chase, in such a way that its speed at  $t$  seconds after having been passed is given by  $s(t)\text{ms}^{-1}$ , where

$$s(t) = \frac{60t}{t + 60}.$$

Show that the police car catches the speeding car within 6 minutes.